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ANALYSIS OF CONDENSATION IN POROUS INSULATION

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Abstract — The present work analyzes the problem of condensation in porous insulation. Specific consideration is given to a steady-state one-dimensional formulation, representing a porous slab exposed to two different humid environments on both sides. The analysis includes both the convective and the diffusive transport mechanisms along with phase change. Condensation (or freezing) is shown to take place in a wet zone in which the air–vapor mixture is saturated. When the two external environments are not saturated, the wet zone is bounded by two dry zones with no condensation. The effect of condensation, due to release of latent heat, on the overall thermal performance is found to be significant. Both the condensation rate and the resulting increase in heat transfer depend on the Peclet number, the Lewis number, and the Biot number, as well as on the temperatures and humidities of the two external environments.

NOMENCLATURE

a_i,	coefficients in temperature distributions, equations (17)–(19), (23);
b_i,	coefficients in vapor concentration distributions, equations (20), (21), (23);
c_i,	coefficients defined by equation (25);
c_p,	constant pressure specific heat $[\mathrm{J\,kg^{-1}\,K^{-1}}]$;
D,	vapor diffusivity $[\mathrm{m^2\,s^{-1}}]$;
G,	overall condensation rate $[\mathrm{kg\,m^{-2}\,s^{-1}}]$;
h,	heat-transfer coefficient $[\mathrm{W\,m^{-2}\,K^{-1}}]$;
l,	length [m];
L,	latent heat of evaporation or sublimation $[\mathrm{J\,kg^{-1}}]$;
Le,	Lewis number (α_0/D_0);
Nu,	Nusselt number (hl/λ_0);
N_R,	ratio of Nusselt number with condensation to the one without condensation (Nu/Nu_d);
P,	pressure $[\mathrm{N\,m^{-2}}]$;
Pe,	Peclet number (ul/α_0);
t,	time [s];
T,	temperature [°C];
u,	cross-flow velocity $[\mathrm{m\,s^{-1}}]$;
W,	vapor concentration $[\mathrm{kg\,kg^{-1}\ air}]$;
x,	coordinate across the porous slab [m].

Greek symbols

α	thermal diffusivity $[\mathrm{m^2\,s^{-1}}]$;
β,	nondimensional parameter defined by equation (10);
Γ,	volumetric condensation (or freezing) rate $[\mathrm{kg\,m^{-3}\,s^{-1}}]$;
η,	vapor saturation gradient with respect to temperature $[\mathrm{K^{-1}}]$;
θ,	liquid saturation $[\mathrm{m^3\,m^{-3}\ void}]$;
λ,	thermal conductivity $[\mathrm{W\,m^{-1}\,K^{-1}}]$;
μ,	relative humidity (W/W_s);
ρ,	density $[\mathrm{kg\,m^{-3}}]$;
Φ,	porosity $[\mathrm{m^3\,m^{-3}}]$.

Subscripts

a,	air or warm environment;
b,	cold environment;
d,	dry (no condensation);
l,	liquid;
o,	porous medium;
s,	saturation;
1, 3,	dry zones;
2,	wet zone.

Superscript

$*$,	nondimensional quantity.

1. INTRODUCTION

WATER-VAPOR condensation within a porous wall insulation has been observed, particularly when exposed to large temperature differences and high humidity environments. Wetting of porous insulation has been found to persist despite the presence of vapor barriers or impermeable close-cell wall coverings. This is attributed to the unavoidable formation of openings arising from faulty installation and aging. The wetting phenomenon is generally believed to cause a significant deterioration in the thermal performance of wall insulation, hence being the subject of major concern [1–4]. However, the existing knowledge on this subject is rather limited. Although some qualitative experimental work has been carried out [1, 4], very little quantitative analysis is presently available [5].

There exists a large body of literature dealing with the general subject of heat and mass transfer in porous medium ranging from general formulation of the

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phenomenon [6–8] to specific applications such as heat and moisture transport in soils [9–13] and in food products [14]. Recently, the problem of transient intensive drying in porous media has been investigated [15–17], and numerical solutions of heat and mass transfer in porous media have been presented [18, 19]. The above is by no means an exhaustive survey of the work on this subject; it is rather a representative sample of the enormous effort in this area. However, as mentioned earlier, very little work exists on the particular subject of condensation in porous insulation. The unique features of this problem are: (1) the convective and diffusive components are equally important, (2) the liquid saturations involved are relatively small, (3) the medium is highly porous and in most instances fibrous rather than the granular or consolidated low porosity materials investigated in the past, and (4) the phenomenon is studied from the standpoint of its resulting effect on the insulating capabilities of the porous medium.

Heat and moisture transport in an installed porous insulation is generally a complex multidimensional problem including air–vapor mixture flow due to free convection and infiltration through openings [20, 21], heat transfer by conduction and convection, vapor transport by diffusion and convection, flow of liquid due to gravity and capillary action (provided the liquid saturation is high enough to render the liquid mobile), and condensation or freezing with a possibility of reevaporation accompanied by release or consumption of latent heat. The solution to this problem is not attempted at this stage; instead, the problem is reduced to a one-dimensional configuration, which still retains, however, most of the important parameters of the original problem.

2. PROBLEM STATEMENT

The one-dimensional configuration investigated here consists of an initially dry porous slab, the boundaries of which are exposed to two different environments: a warm environment with temperature T_a, vapor concentration W_a, and pressure P_a, and a colder environment with temperature T_b, vapor concentration W_b, and pressure P_b. Cross flow of air and water-vapor mixture with velocity u is established due to the pressure difference across the porous slab (Fig. 1). Velocities of magnitudes similar to those encountered in porous insulation due to free convection and infiltration are considered. These velocities are very small (of the order 10^{-3} m s^{-1}), hence rendering both the convective and the diffusive transport mechanisms equally important.

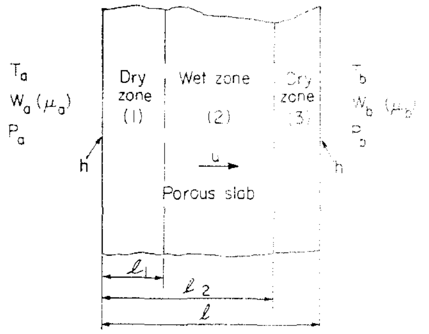

FIG. 1. One-dimensional porous slab exposed to two humid environments.

The dry porous slab is expected to exhibit three stages in its response to the two external humid environments. While these three stages are actually segments of a continuous process and do overlap, it is convenient to consider them separately. First occurs a relatively short initial transient stage in which the temperature and vapor concentration fields are developing within the porous slab. During this process a very small quantity of liquid water, mostly in a bound form, is accumulated in the porous medium. This transient liquid accumulation is governed by the well-known Kelvin relation [9]. This relation describes the equilibrium liquid saturation in the presence of vapor at a given temperature and vapor concentration, taking into account the capillary or adsorption potential at the interface between the liquid and gaseous phases. Condensation is defined here as the accumulation of liquid beyond the adsorption process described above. The region where the air–vapor mixture is saturated and condensation, as defined here, occurs is referred to as a wet zone. In the transient stage the wet zone is being established along with the temperature and vapor concentration distributions. In the second stage, a steady-state situation prevails and the temperature and vapor concentration fields are invariant in time. This situation is maintained so long as the liquid content, which does vary in time, is small enough as to have an insignificant effect on properties, and provided the liquid is in a pendular state which renders it practically immobile. Under these conditions the wet zone, in which liquid is accumulated at a constant rate, is stationary. When the two external environments are not saturated, two dry zones will exist adjacent to the two boundaries, as shown in Fig. 1. However, if one of the environments is saturated, the wet zone will extend to the corresponding boundary. In a dry zone the very small quantity of adsorbed liquid, corresponding to the equilibrium condition described by the Kelvin relation, remains unchanged. Finally, a transient stage follows when the liquid saturation reaches high enough values as to affect the properties significantly. As the liquid content increases further, the liquid starts to flow out of the wet zone by capillary action. In this stage the distributions are time dependent and the wet zone expands due to the liquid outflow. The condensation rate decreases in time due to reevaporation occurring in the expanded wet region and due to a decrease in cross flow velocity caused by the additional impedance presented by the liquid in the porous medium.

The initial transient stage is of little significance due to its relatively short duration and therefore is not studied here. Furthermore, due to the very small rate of condensation occurring during the steady-state situation (as is demonstrated in the solution to follow), the second stage will be maintained over a relatively long period of time. The present study is confined to the investigation of the steady-state situation described above; the latter transient stage will be the subject of future work.

Before analyzing condensation, it must be established whether, for the conditions considered, condensation indeed takes place. This is done by following a procedure similar in principle to Takashi's [1, 5]. Solutions for both the temperature and vapor concentration distributions are obtained neglecting condensation (these solutions are presented later as special cases of the general analysis). If at any point in the porous slab the vapor concentration, obtained from this dry solution, is higher than the saturation value corresponding to the temperature at that point, condensation occurs and the analysis which includes condensation must be followed. Otherwise, condensation does not take place and the simple dry solution is the solution to the problem.

The following analysis applies to both condensation and freezing. However, the latent heat of sublimation must replace the latent heat of evaporation in the region where freezing occurs.

3. MATHEMATICAL FORMULATION

The following assumptions and restrictions, of which some were mentioned earlier, are invoked in the present formulation:

(1) Local thermal equilibrium exists among all phases due to the relatively low velocities considered and the small pore dimensions of the medium. Therefore, all phases have an identical temperature at a point in space, and the air–vapor mixture is at saturation conditions in the presence of liquid.

(2) The liquid is immobile and its effects on the properties is negligible. Both of these assumptions are reasonable for small liquid saturations.

(3) The initial adsorption process is not taken into consideration in light of its very small contribution to the overall liquid content; only the liquid accumulation due to condensation in the wet zone is investigated.

(4) The properties of the gas phase are taken to be those of dry air because the vapor concentration is rather low (less than 5%) at the temperatures considered here.

(5) All the properties are independent of temperature in view of the relatively small temperature differences studied (less than or equal to 40°C).

(6) The transfer coefficients on the slab boundaries are known, and the Lewis number outside the porous slab is approximately unity.

(7) Cross-flow velocities are small (of the order $10^{-3}\,\mathrm{m\,s^{-1}}$). Hence both convective and diffusive components of transport mechanisms are included.

(8) Free convection in the porous slab is negligible.

Based on the above assumptions and restrictions, the conservation equations for energy, vapor, and liquid, respectively, are:

$$(\rho c_p)_a u \frac{\mathrm{d}T}{\mathrm{d}x} = \lambda_0 \frac{\mathrm{d}^2 T}{\mathrm{d}x^2} + L\Gamma \tag{1}$$

$$\rho_a u \frac{\mathrm{d}W}{\mathrm{d}x} = \rho_a D_0 \frac{\mathrm{d}^2 W}{\mathrm{d}x^2} - \Gamma \tag{2}$$

$$\rho_l \Phi \frac{\partial \theta}{\partial t} = \Gamma. \tag{3}$$

In addition, both the condensation rate Γ and the accumulated liquid saturation θ are identically zero in a dry zone, whereas in a wet zone the vapor concentration is given by the saturation relation which is a sole function of temperature, $W = W_s(T)$. The boundary conditions, invoking $Le = 1$ outside the slab, are:

$$\begin{aligned} x = 0:\quad & h[T_a - T(0)] = -\lambda_0 \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_0 ; \\ & \frac{h}{(\rho c_p)_a}[W_a - W(0)] = -D_0 \left.\frac{\mathrm{d}W}{\mathrm{d}x}\right|_0 \\ x = l:\quad & h[T(l) - T_b] = -\lambda_0 \left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_l ; \\ & \frac{h}{(\rho c_p)_a}[W(l) - W_b] = -D_0 \left.\frac{\mathrm{d}W}{\mathrm{d}x}\right|_l . \end{aligned} \tag{4}$$

All the symbols in the above equations are defined in the Nomenclature.

The variables are nondimensionalized as follows:

$$x^* = \frac{x}{l}; \quad T^* = \frac{T - T_b}{T_a - T_b}; \quad W^* = \frac{W - W_b}{W_a - W_b}. \tag{5}$$

The resulting nondimensional equations and boundary conditions are

$$Pe \frac{\mathrm{d}T^*}{\mathrm{d}x^*} = \frac{\mathrm{d}^2 T^*}{\mathrm{d}x^{*2}} + \frac{\beta}{Le}\Gamma^* \tag{6}$$

$$Le\,Pe \frac{\mathrm{d}W^*}{\mathrm{d}x^*} = \frac{\mathrm{d}^2 W^*}{\mathrm{d}x^{*2}} - \Gamma^* \tag{7}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t^*} = \Gamma^* \tag{8}$$

$$\begin{aligned} x^* = 0:\quad & Bi[1 - T^*(0)] = -\left.\frac{\mathrm{d}T^*}{\mathrm{d}x^*}\right|_0 ; \\ & Le\,Bi[1 - W^*(0)] = -\left.\frac{\mathrm{d}W^*}{\mathrm{d}x^*}\right|_0 \\ x^* = 1:\quad & Bi\,T^*(1) = -\left.\frac{\mathrm{d}T^*}{\mathrm{d}x^*}\right|_1 ; \end{aligned} \tag{9}$$

$$Le\,Bi\,W^*(1) = -\left.\frac{\mathrm{d}W^*}{\mathrm{d}x^*}\right|_1$$

where

$$Pe = ul/\alpha_0;\quad Le = \alpha_0/D_0;\quad Bi = hl/\lambda_0$$

$$\alpha_0 = \lambda_0/(\rho c_p)_a;\quad \beta = L(W_a - W_b)/c_p(T_a - T_b) \qquad (10)$$

$$\Gamma^* = \Gamma l^2/D_0\rho_a(W_a - W_b);\; t^* = tD_0\rho_a(W_a - W_b)/\Phi\rho l^2.$$

Since in the dry zones (designated as 1 and 3), the liquid saturation and the volumetric condensation rates are identically zero, the above equations reduce to the following:

$$Pe\frac{\mathrm{d}T^*_{1,3}}{\mathrm{d}x^*} = \frac{\mathrm{d}^2T^*_{1,3}}{\mathrm{d}x^{*2}} \qquad (11)$$

$$Le\,Pe\frac{\mathrm{d}W^*_{1,3}}{\mathrm{d}x^*} = \frac{\mathrm{d}^2W^*_{1,3}}{\mathrm{d}x^{*2}}. \qquad (12)$$

In the wet zone (designated as 2), the vapor concentration is a singular function of temperature, $W^* = W^*_s(T)$. Therefore, equations (6) and (7) can be combined, by eliminating the source term between them, into the following single equation in T^*:

$$Le\,Pe[1 + \beta\eta^*(T_2)]\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*} = \frac{\mathrm{d}}{\mathrm{d}x^*}[Le + \beta\eta^*(T_2)]\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*} \qquad (13)$$

where

$$\eta(T) = \frac{\mathrm{d}W_s(T)}{\mathrm{d}T} \quad\text{and}\quad \eta^*(T) = \frac{T_a - T_b}{W_a - W_b}\eta(T). \qquad (14)$$

Transforming equation (7) into an equation in T^* and substituting for the source term in equation (8), there results

$$\frac{\partial\theta_2}{\partial t^*} = \frac{\mathrm{d}}{\mathrm{d}x^*}\left[\eta^*(T_2)\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*}\right] - Le\,Pe\,\eta^*(T_2)\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*} = \Gamma^*. \qquad (15)$$

Continuity of both the temperature and vapor concentration distributions as well as the continuity of their gradients at the interfaces between the dry and wet zones constitutes a set of eight interface conditions in addition to the four boundary conditions stated earlier, equation (9). The nondimensional form of these interface and boundary conditions is:

$$x^* = 0:\quad Bi[1 - T^*_1(0)] = -\left.\frac{\mathrm{d}T^*_1}{\mathrm{d}x^*}\right|_0;$$

$$Le\,Bi[1 - W^*_1(0)] = -\left.\frac{\mathrm{d}W^*_1}{\mathrm{d}x^*}\right|_0 \qquad (16a)$$

$$x^* = l^*_1:\quad T^*_1(l^*_1) = T^*_2(l^*_1);\quad W^*_1(l^*_1) = W^*_s[T_2(l^*_1)] \qquad (16b)$$

$$\left.\frac{\mathrm{d}T^*_1}{\mathrm{d}x^*}\right|_{l^*_1} = \left.\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*}\right|_{l^*_1};\quad \left.\frac{\mathrm{d}W^*_1}{\mathrm{d}x^*}\right|_{l^*_1} = \eta^*[T_2(l^*_1)]\left.\frac{\mathrm{d}T^*}{\mathrm{d}x^*}\right|_{l^*_1} \qquad (16c)$$

$$x^* = l^*_2:\; T^*_2(l^*_2) = T^*_3(l^*_2);\quad W^*_s[T_2(l^*_2)] = W^*_3(l^*_2) \qquad (16d)$$

$$\left.\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*}\right|_{l^*_2} = \left.\frac{\mathrm{d}T^*_3}{\mathrm{d}x^*}\right|_{l^*_2};\quad \eta^*[T_2(l^*_2)]\left.\frac{\mathrm{d}T^*_2}{\mathrm{d}x^*}\right|_{l^*_2} = \left.\frac{\mathrm{d}W^*_3}{\mathrm{d}x^*}\right|_{l^*_2} \qquad (16e)$$

$$x^* = 1:\; Bi\,T^*_3(1) = -\left.\frac{\mathrm{d}T^*_3}{\mathrm{d}x^*}\right|_1;$$

$$Le\,Bi\,W^*_3(1) = -\left.\frac{\mathrm{d}W^*_3}{\mathrm{d}x^*}\right|_1. \qquad (16f)$$

4. SOLUTION

The above formulation constitutes a set of five differential equations for the five unknown distributions $T^*_1(x^*)$, $T^*_2(x^*)$, $T^*_3(x^*)$, $W^*_1(x^*)$, and $W^*_3(x^*)$: two linear equations, (11) and (12), for each of the dry zones, and a single nonlinear equation, (13), for the wet zone. The vapor concentration distribution in the wet zone is the saturation distribution corresponding to the temperature field, $W^*_2(x^*) = W^*_s[T_2(x^*)]$. The liquid accumulation can be found, after the solution for temperature and vapor concentration has been obtained, with the use of the temperature solution in the wet zone, equation (15). There are altogether twelve boundary and interface conditions given by equation (16). However, the five second-order differential equations require only ten of these conditions, therefore the remaining two serve to locate the two interfaces l^*_1 and l^*_2.

The differential equations are integrated twice, incorporating the four boundary conditions, equations (16a) and (16f), to yield the following expressions

$$T^*_1(x^*) = a_1[\exp(Pe\,x^*) - (1 - Pe/Bi)] + 1 \qquad (17)$$

$$T^*_2(x^*) = a_3\int_{l^*_1}^{x^*}\left\{\exp\left[\int_{l^*_1}^{\zeta} f(v)\mathrm{d}v\right]\right\}\mathrm{d}\zeta + a_4 \qquad (18)$$

$$T^*_3(x^*) = a_5[\exp(Pe\,x^*) - \exp(Pe)(1 + Pe/Bi)] \qquad (19)$$

$$W^*_1(x^*) = b_1[\exp(Le\,Pe\,x^*) - (1 - Pe/Bi)] + 1 \qquad (20)$$

$$W^*_3(x^*) = b_5[\exp(Le\,Pe\,x^*) - \exp(Le\,Pe)(1 + Pe/Bi)] \qquad (21)$$

where

$$f(x) = \frac{Le\,Pe[1 + \beta\eta^*(x)] - \beta\,\mathrm{d}\eta^*(x)/\mathrm{d}x}{Le + \beta\eta^*(x)};$$

$$\eta^*(x) = \eta^*[T_2(x)]. \qquad (22)$$

The six coefficients in the above expressions (a_is) and

b_is) are determined from six interface conditions, equations (16b)–(16e), to yield

$$\begin{aligned} a_1 &= 1/[c_2(c_4c_5/c_6 - c_1/c_2 - c_3)]; \\ a_3 &= 1/(c_4c_5/c_6 - c_1/c_2 - c_3) \\ a_4 &= 1 + c_1/[c_4c_5/c_6 - c_1/c_2 - c_3)]; \\ a_5 &= c_5/[c_6(c_4c_5/c_6 - c_1/c_2 - c_3)] \\ b_1 &= \{W_s^*[T_2(l_1^*)] - 1\}/c_7; \\ b_5 &= W_s^*[T_2(l_2^*)]/c_8. \end{aligned} \tag{23}$$

The locations of the two interfaces, l_1^* and l_2^*, are obtained via the remaining two interface conditions for continuity of vapor concentration gradients across the interface, equations (16c) and (16e). These conditions can be rewritten as

$$b_1 c_9 = a_3 \eta^*(l_1^*); \quad a_3 c_5 \eta^*(l_2^*) = b_5 c_{10}. \tag{24}$$

Coefficients c_1–c_{10} are defined as follows:

$$\begin{aligned} c_1 &= \exp(Pe\, l_1^*) - (1 - Pe/Bi); \\ c_2 &= Pe \exp(Pe\, l_1^*) \\ c_3 &= \int_{l_1^*}^{l_2^*} \left\{ \exp\left[\int_{l_1^*}^{\zeta} f(v) \right] \mathrm{d}v \right\} \mathrm{d}\zeta \\ c_4 &= \exp(Pe\, l_2^*) - (1 + Pe/Bi)\exp(Pe) \\ c_5 &= \exp\left[\int_{l_1^*}^{l_2^*} f(v)\,\mathrm{d}v \right]; \quad c_6 = Pe \exp(Pe\, l_2^*) \\ c_7 &= \exp(Le\, Pe\, l_1^*) - (1 - Pe/Bi) \\ c_8 &= \exp(Le\, Pe\, l_2^*) - (1 + Pe/Bi)\exp(Le\, Pe) \\ c_9 &= Le\, Pe \exp(Le\, Pe\, l_1^*); \\ c_{10} &= Le\, Pe \exp(Le\, Pe\, l_2^*). \end{aligned} \tag{25}$$

To generate solutions from the above expressions, a numerical code was developed. This code consists of two iteration procedures, one within the other. The inner one involves a successive substitution into equation (18) in conjunction with equations (22), (23) and (25), in order to solve the nonlinear equation for the temperature distribution in the wet zone $T_2^*(x^*)$. The outer iteration procedure searches for a combination of the interface locations, l_1^* and l_2^*, which satisfy equation (25).

For the case of no condensation in the porous slab, the dry solution for temperature and vapor concentration are obtained analytically from equations (12) and (13), which are now valid for the entire slab. When the boundary conditions, equations (17a) and (17f), are incorporated, the following solution for temperature and vapor concentration distribution results

$$T_d^*(x^*) = \frac{\exp(Pe\, x^*) - (1 + Pe/Bi)\exp(Pe)}{1 - Pe/Bi - (1 + Pe/Bi)\exp(Pe)} \tag{26}$$

$$W_d^*(x^*) = \frac{\exp(Le\, Pe\, x^*) - (1 + Pe/Bi)\exp(Le\, Pe)}{1 - Pe/Bi - (1 + Pe/Bi)\exp(Le\, Pe)}. \tag{27}$$

As discussed earlier, these dry solutions may be used to establish whether condensation occurs. If $W_d^*(x^*)$ is higher than $W_s^*[T_d^*(x^*)]$ at any point in the slab, condensation does take place and the formulation which includes condensation must be used.

5. RESULTS AND DISCUSSION

In order to demonstrate the effects of the various parameters as well as the boundary conditions on the rate of condensation and the resulting thermal performance, a variety of cases have been investigated and are presented here. The properties and the external conditions were chosen to simulate typical porous wall insulations. For the purpose of illustration, one such set of properties and conditions, along with the resulting nondimensional parameters and variables, is given below

$$\begin{aligned} \rho_a &= 1.2\,\mathrm{kg\,m^{-3}}; \\ c_{pa} &= 1000\,\mathrm{J^{-1}\,kg}; \\ D_a &= 2.5 \times 10^{-5}\,\mathrm{m\,s^{-2}}; \\ \Phi &= 0.98; \\ D_0 &\simeq \Phi D_a \simeq 2.45 \times 10^{-5}\,\mathrm{m\,s^{-2}}; \\ \lambda_0 &= 0.05\,\mathrm{W\,m^{-1}\,K^{-1}}; \\ \alpha_0 &= \lambda_0/(\rho c_p)_a \simeq 4.0 \times 10^{-5}\,\mathrm{m\,s^{-2}}; \\ L(\text{evap.}) &= 2.5 \times 10^6\,\mathrm{J\,kg^{-1}}; \\ L(\text{sublim.}) &= 2.8 \times 10^6\,\mathrm{J\,kg^{-1}}; \\ l &= 0.15\,\mathrm{m};\ u = 5 \times 10^{-4}\,\mathrm{m\,s^{-1}}; \\ h &= 10\,\mathrm{W\,m^{-2}\,K^{-1}}; \\ T_a &= 30^\circ\mathrm{C};\ T_b = -10^\circ\mathrm{C}; \\ \mu_a &= \mu_b = 0.8\,(\mu = W/W_s). \end{aligned}$$

And the resulting nondimensional parameters are

$$\begin{aligned} & Le = \alpha_0/D_0 \simeq 1.7; \quad Pe = ul/\alpha_0 \simeq 1.8; \\ & Bi = hl/\lambda_0 \simeq 30 \\ & \beta = L(W_a - W_b)/c_{pa}(T_a - T_b) \simeq 1.4 \\ & \eta^*(10^\circ\mathrm{C}) = \eta(10^\circ\mathrm{C})(T_a - T_b)/(W_a - W_b) \simeq 0.6 \\ & \Gamma^* = \Gamma l^2/D_0 \rho_a (W_a - W_b) \simeq 3.7 \times 10^4\, \Gamma \\ & t^* = t D_0 \rho_a (W_a - W_b)/\Phi \rho_l l^2 \simeq 2.8 \times 10^{-8}\, t. \end{aligned}$$

The dry distributions W_d^* and $W_{s,d}^*$ (which neglect condensation in the porous slab), along with the wet distributions W^*, W_s^* and Γ^* (which include the effect of condensation), are depicted in Fig. 2. The dry vapor distribution W_d^* crosses over the saturation distribution $W_{s,d}^*$, indicating the presence of condensation. The difference between the dry and wet distributions appears to be significant. Furthermore, the presence of the three zones discussed earlier is clearly demonstrated. Incidentally, T_m denotes the average of the two external temperatures T_a and T_b, while ΔT denotes the difference between the two.

Figures 3 and 4 compare distributions of vapor concentration and temperature for various external humidities with the corresponding dry curves. The deviations from the dry curves are larger for higher external humidities due to higher condensation rates. Since volumetric condensation is a distributed sink for

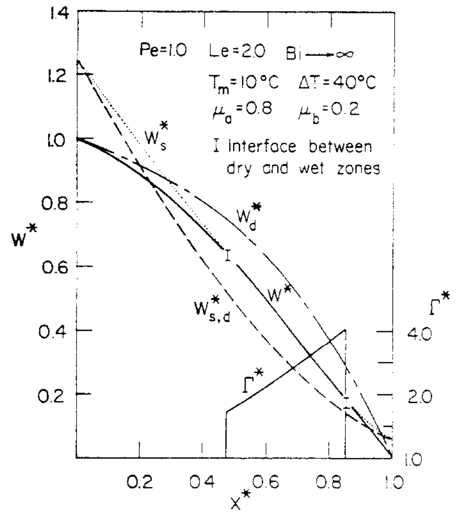

FIG. 2. Dry and wet distributions of vapor concentrations, saturation, and volumetric condensation.

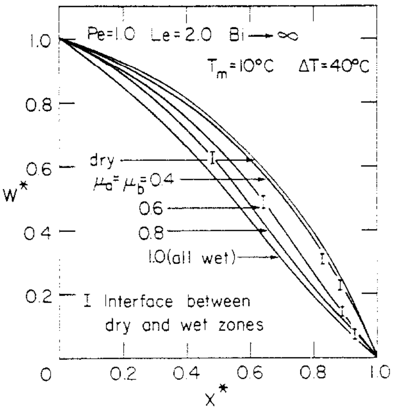

FIG. 3. Vapor concentration distributions for various external humidities.

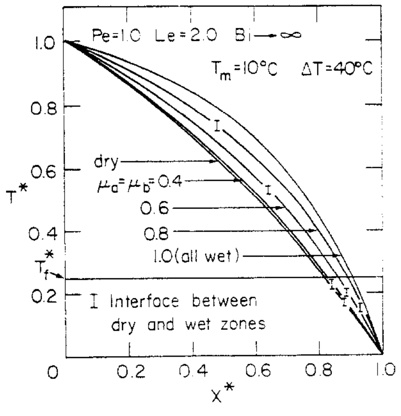

FIG. 4. Temperature distributions for various external humidities.

vapor concentration but a source for temperature, the vapor concentration curves fall below the dry curve, whereas the temperature curves fall above it. For the particular temperature levels selected here, freezing is shown to occur since portions of the wet zones are below the freezing temperature denoted in Fig. 4 by T_f^*.

The overall nondimensional condensation rate in the slab G^* is plotted in Figs. 5–7. Its definition in terms of the volumetric condensation rate is

$$G^* = \int_0^1 \Gamma^* \mathrm{d}x^* = \int_{l_1^*}^{l_2^*} \Gamma^* \mathrm{d}x^*. \tag{28}$$

In addition, to demonstrate the trends of the dimensional condensation rate, the product $G^*(W_a - W_b)$ is also presented whenever the overall vapor concentration difference $(W_a - W_b)$ is not constant. This product arises from the nondimensionalization of the source term Γ, equation (10). The solution for relatively large convection ($Pe = 10$) and small Biot number ($Bi = 10$) could not be attained because, under these conditions, the wet zone is 'blown out' of the slab to the downstream boundary layer, a situation that cannot be handled by the present formulation.

Most of the trends observed above can be explained in physical terms. It is postulated here that the larger the crossover of the dry vapor concentration distribution W_d^* and the saturation distribution $W_{s,d}^*$ (an example of which is shown in Fig. 2), the higher the condensation rate will become. This crossover occurs due to the two following reasons: the saturation relation $W_s(T)$ is nonlinear in T, and the deviation from the linear distribution due to convection ($Pe > 0$) is larger for vapor concentration than for temperature at Lewis numbers greater than unity. In addition, the condensation rate is generally larger for higher vapor

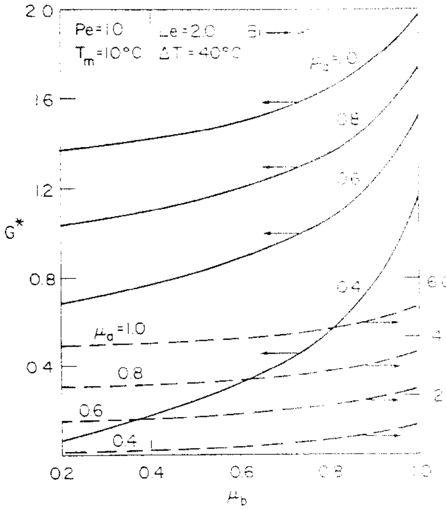

FIG. 5. Overall condensation rates for various external humidities.

mass fluxes as well as higher vapor concentrations.

Figure 5 demonstrates an expected increase in dimensional and nondimensional condensation rate with the external humidities and more so with the humidity on the warm side due to increased crossover as well as higher levels of vapor concentrations. For higher overall temperature differences ΔT and lower temperature levels T_m, the nonlinearity in $W_s(T)$ is more pronounced, causing a larger crossover which results in higher nondimensional condensation rates (Fig. 6). However, due to the rapid increase in vapor concentration levels with temperature, the dimensional condensation rate, which is proportional to $G^*(W_a - W_a)$, exhibits an opposite trend with respect to temperature levels. The rate of condensation increases considerably with the Peclet number (Fig. 7), due to the increased vapor mass flux. Since the crossover becomes larger as the Lewis number increases, so does the condensation rate. The smaller condensation rates observed for $Bi = 10$, in comparison with $Bi \to \infty$, are due to a smaller effective temperature difference across the slab for the same external temperatures.

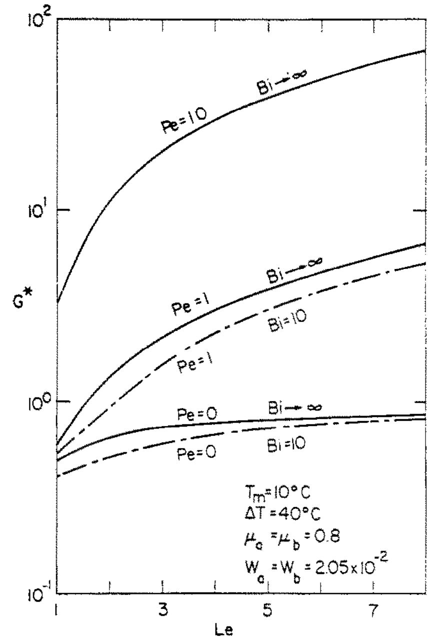

FIG. 7. Overall condensation rates for various Peclet, Lewis, and Biot numbers.

The effect of condensation on the overall heat-transfer coefficient is represented by the ratio of Nusselt number with condensation to the one without, $N_R = Nu/Nu_d$, and is plotted in Figs. 8, 9, and 10. The Nusselt number is defined as the ratio of the sensible heat crossing the cold boundary by conduction and convection to the heat flux by conduction alone. The higher the ratio of the Nusselt numbers, the more severe is the deterioration in the insulating performance of the porous slab due to condensation. This effect is generally proportional to the rate of condensation as can be seen by the similarity in trends between Figs. 8–10 and Figs. 5–7. The only exception occurs in the case of $Pe = 0$ (Fig. 10), where the moderate increase in condensation rate with the Lewis number exhibited in Fig. 7 is accompanied by a decrease of the Nusselt-number ratio. This is attributed to the proportionality of the source term in the energy equation (6), to the inverse of the Lewis number. Therefore, if the condensation rate increases slower than the Lewis number, this source term will actually decrease.

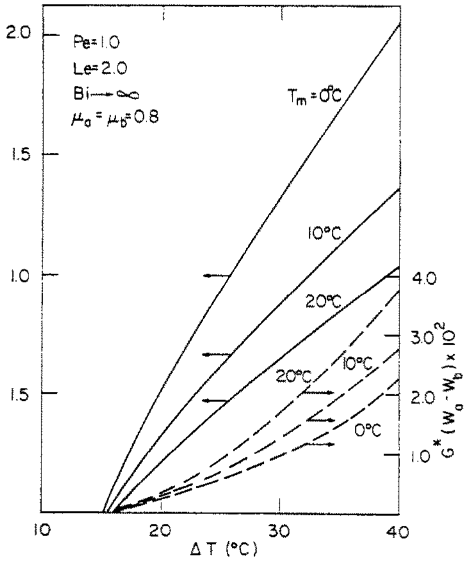

FIG. 6. Overall condensation rates for various mean temperatures and temperature differences.

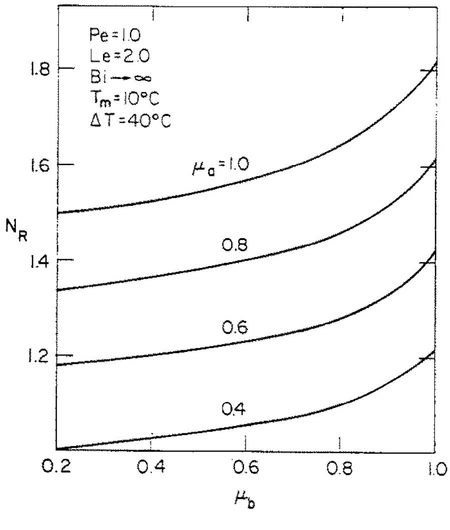

FIG. 8. Effect of condensation on the Nusselt number for various external humidities.

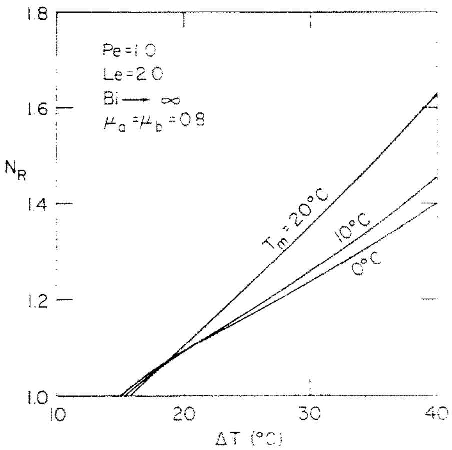

FIG. 9. Effect of condensation on the Nusselt number for various external temperatures.

6. CONCLUDING REMARKS

A steady one-dimensional solution is presented for the problem of condensation in porous insulation due to diffusion and convection of vapor. The existence of three zones, one dry and two wet, is demonstrated. Despite a relatively small rate of condensation, a significant effect is observed on the thermal performance of insulation due to the release of latent heat within the porous slab. The condensation rate and the resulting increase in overall heat transfer is found to increase with external humidities, temperature levels, and overall temperature differences. The effects of the porous medium properties, such as thermal diffusivity, vapor diffusivity and thermal conductivity, as well as its width in addition to the cross flow velocity and external heat-transfer coefficients, are demonstrated via the following three nondimensional parameters: the Peclet number, the Lewis number and the Biot number. Both the condensation rate and the overall heat transfer generally increase with the above three parameters.

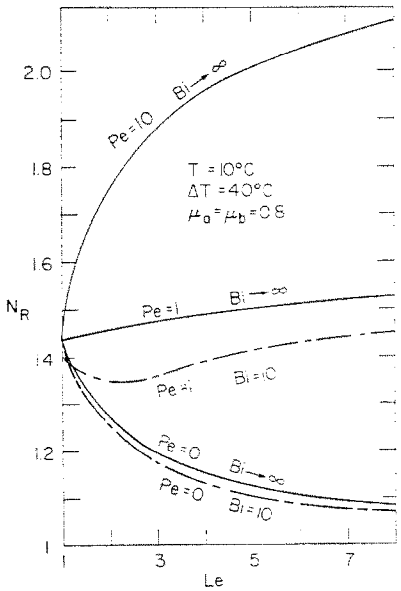

FIG. 10. Effect of condensation on the Nusselt number for various Peclet, Lewis, and Biot numbers.

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ANALYSE DE LA CONDENSATION DANS LES ISOLANTS POREUX

Résumé—Cet article analyse le problème de la condensation dans les isolants poreux. On considère le problème unidirectionnel et permanent d'une plaque poreuse exposée à deux environnements différents d'humidité sur ses deux faces. L'analyse inclut les mécanismes de transport par convection et diffusion avec changement de phase. La condensation (ou le gel) prend place dans une zone humide où le mélange air–vapeur d'eau est saturé. Quand les deux environnements ne sont pas saturés, la zone humide est bordée par deux zones sèches sans condensation. L'effet de la condensation due à la cession de chaleur latente, sur les performances thermiques globales est très sensible. Le taux de condensation et l'accroissement résultant de transfert thermique dépendent des nombres de Peclet, de Lewis et de Biot aussi bien que des températures et des humidités des deux environnements.

UNTERSUCHUNG DER KONDENSATION IN PORÖSER ISOLIERUNG

Zusammenfassung — In der vorliegenden Arbeit wird das Problem der Kondensation in einer porösen Isolierung untersucht. Insbesondere wird der stationäre eindimensionale Fall behandelt, den eine poröse Platte darstellt, welche an beiden Seiten einer unterschiedlich feuchten Umgebung ausgesetzt ist. Die Untersuchung schließt sowohl den Konvektions- als auch den Diffusions-Transportvorgang bei gleichzeitigem Phasenwechsel ein. Es wird gezeigt, daß Kondensation (oder Ausfrieren) in einem feuchten Gebiet bei Sättigung des Luft-Wasserdampf-Gemisches stattfindet. Wenn die zwei äußeren Umgebungen nicht gesättigt sind, ist die feuchte Zone durch zwei trockene Gebiete ohne Kondensation begrenzt. Es zeigt sich, daß die Auswirkung der Kondensation auf das thermische Gesamtverhalten durch das Freiwerden von latenter Wärme bedeutend ist. Sowohl das Ausmaß der Kondensation als auch der daraus folgende Anstieg des Wärmeübergangs hängen von der Peclet-Zahl, der Lewis-Zahl und der Biot-Zahl ab, ebenso von den Temperaturen und Feuchtigkeiten der beiden äußeren Umgebungen.

АНАЛИЗ ПРОЦЕССА КОНДЕНСАЦИИ В ПОРИСТОЙ ИЗОЛЯЦИИ

Аннотация — Проведен анализ процесса конденсации в пористой изоляции. Особое внимание уделено стационарному одномерному случаю, когда рассматривается пористая плита, по обеим сторонам которой окружающая среда имеет различную влажность. Наряду с фазовыми переходами учитываются как конвективный, так и диффузионный механизмы переноса. Показано, что конденсация (или замораживание) происходит во влажной зоне, где смесь пара с воздухом является насыщенной. Если внешняя среда по обе стороны плиты не насыщена, влажная зона ограничена двумя сухими, где конденсация отсутствует. Найдено, что за счет выделения скрытой теплоты процесс конденсации существенным образом влияет на суммарные тепловые характеристики. Скорость конденсации и интенсификация теплопереноса зависит от чисел Пекле, Льюиса и Био, а также от температуры и влажности окружающей среды.